## St. Loys C of E Primary Academy

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## INTRODUCTION

Mathematics is a creative and highly interconnected discipline that has been developed over centuries. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

## MASTERY IS FOR ALL.....AIMS OF THE POLICY

- To ensure all children leave our school with a secure understanding of the four operations and can confidently use both written and mental calculation strategies in a range of contexts.
- To ensure that consistent strategies, models and images are utilised across the school to embed and deepen children's learning and understanding of mathematical concepts.
- To inform and further strengthen the links between school and home learning opportunities giving all stakeholders clarity on how St Loys approaches mathematical learning.


## HOW TO USE THIS POLICY

This policy has been designed to support the teaching and planning of mathematics in our school. The policy only details the strategies, and teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum. The examples and illustrations are not exhaustive but provide an overall picture of what the mathematics in our school should look like. This is not a scheme of work and must be used in conjunction with Evolve MAT Mathematics Policy and curriculum documents.
This policy sets out the progression of strategies and written methods which children will be taught as they develop in their understanding of the four operations. Strategies refer to a Concrete, Pictorial, Abstract (CPA) approach to develop children's deep understanding and mastery of mathematical concepts.
Children use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters, cubes or Numicon. This is then developed through the use of images, models and children's own pictorial representations before moving on to the abstract mathematics. Children will travel along this continuum again and again, often revisiting previous stages when a concept is extended. It is also worth noting that if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something' or 'working out' could involve use of the concrete or pictorial.
Similarly, although the strategies are taught in a progressive sequence, they are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. So as a new strategy is taught it does not necessarily supersede the previous, but builds on prior learning to enable children to have a variety of tools to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies that are most efficient for the task.
The strategies are separated into the 4 operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be referred to/taught together to ensure that children are making connections and seeing relationships in their mathematics.

Therefore, where appropriate some strategies will be taught simultaneously, for example, counting on (addition) and counting back (subtraction). Effective teaching of these strategies relies upon increasing levels of number sense, fluency and the ability to reason mathematically.
Children will move through the strategies at a pace appropriate to the age related expectations as defined in the EYFS and NC, respectively.

## Developing understanding of the $=$ symbol

It is important that when teaching the 4 operations children's understanding of the $=$ symbol is developed. The symbol = is an assertion of equivalence as in the number sentence:

$$
3+4=6+1
$$

Misconceptions that the = symbol only means "work out the answer to this calculation" can be confusing when children are presented with empty box questions such as:

$$
3+\square=8
$$

Therefore teachers should present children with number sentences and problems that place the $=$ symbol in different positions and in different contexts. In the concrete phase balance scales and Numicon provide a useful resource to demonstrate equality.

## Teaching inequality alongside teaching equality

To help children develop their understanding of equality, they also need to develop understanding of inequality. One way to introduce the < and > signs is to use rods and cubes to make concrete and visual representations such as:

to show that 5 is greater than $2(5>2), 5$ is equal to $5(5=5)$, and 2 is less than 5 (2 < 5)
Balance scales can also be used to represent inequality.
Incorporating both equality and inequality into examples and exercises can help children to develop their conceptual understanding:

$$
5+7 \square 5+6
$$

....their mathematical reasoning: 'I know that 7 is greater than 6 , so 5 plus 7 must be greater than 5 plus 6"

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology and to explain their mathematical thinking in complete sentences.

## Stages in Addition

## Teaching for Mastery FS/KS1 example CPA

| Strategies | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Combining 2 groups to make a whole Counting sets of objects, recounting using recounting using correspondence. |  |  | $4+3=7$ <br> I have 4 apples and I pick 3 more, how many have I got altogether? <br> Mental recall of number bonds $\begin{aligned} & 6+4=10 \\ & \square+3=15 \quad 19+\square=20 \end{aligned}$ |

## Addition - Foundation Stage

- Say a number which is one more than a given number
- Use quantities and objects to add 2 single digit numbers by counting on

Children will engage in a wide variety of songs and rhymes, games and activities involving counting on in ones, twos, fives and tens. They will begin to relate addition to combing two groups of objects, first by counting all using 1:1 correspondence and then by counting on from the largest number.
In practical activities, using concrete resources and through discussion, they will begin to use the vocabulary involved in addition. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together/we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5 . We have got 5 altogether". Adults support children in recording their addition sums in the written form on whiteboards and in their maths books.


Addition - Year One

- Given a number, identify one more
- Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign
- Add one-digit and two-digit numbers within 20, including zero
- Solve missing number problems eg $10+\square=16$

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practice counting on from any number e.g. 'Put five in your head and count on four.'
Utilising a number track and number lines to count on for addition, counting on from the largest number:

$$
1123456789(0
$$

$5+4=9$
'Put your finger on number five. Count on (count forwards) four.'

With progression to a marked number line:
$6+6=12$

'Put your finger on number six and count on six'
$8+7=15$ 'put your finger on number eight and count on seven.'


Ensure children are confident with using a marked number line before moving on to using an empty number line (see year 2 guidance).

Continue to practice counting on from the largest number for addition with totals within 20.

Teaching for Mastery FS/upper KS1 example CPA

Adding 3
single digits
Use this method as an opportunity to develop fluent recall and
application of known number facts including bonds and doubles.

(2)
$6+3+4=13$

## Addition - Year Two

- Add numbers using concrete objects, pictorial representations, and mentally, including:
- A two digit number and ones
- A two digit number and tens
- Two two-digit numbers
- Three one- digit numbers

NB Ensure that children are confident with the methods outline in the pervious year's guidance before moving on.

Counting on in ones using an empty number line, within 100....
$28+6=34$


And in tens
$28+30=58$


Use in conjunction with a 100 square to show jumps of tens.
$48+36=84$
'Put the biggest number first (48) and then partition the smallest number ( $36=30$ and 6 ) and count on: $48+30+6$


Use in conjunction with a 100 square to show jumps of tens and ones.

If children are confident, use more efficient jumps....


Use in conjunction with a 100 square to show jumps of tens and ones/units.

Also use the partitioning method to add two two-digit numbers:


$$
\begin{gathered}
40+20=60 \\
3+5=8 \\
60+8=68
\end{gathered}
$$

'Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together.
Recombine to give the answer'.


Then move on to calculations that bridge the tens:

$$
\begin{gathered}
48+36=40+8+30+6 \\
40+30=70 \\
8+6=14 \\
70+14=84 \\
48+36=84
\end{gathered}
$$

This is an alterative way of recording the partitioning method.
Further develop addition with numbers that bridge 100, using a 200 grid to support.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year 3

- Add numbers with up to three digits, using formal written methods of columnar addition

NB Ensure that children are confident with the methods outlines in the previous year's guidance before moving on.
$78+46=124$


Use a 200 grid to support counting on in tens and bridging 100 ....
... and with addition of a three-digit and a two-digit number:
$165+56=221$


Further develop the partition method with calculations that bridge 100

```
85+37=80+5+30+7
```

$$
\begin{gathered}
80+30=110 \\
5+7=12 \\
110+12=122
\end{gathered}
$$

$$
85+37=122
$$

The partitioning methods can also be used with three- digit numbers.

Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it has not been necessary to bridge across the tens or hundreds:
$63+32=95$
'Partition the numbers into tens and
$60+3$ ones/units. Add the tens together and
$+30+2$
$90+5=95$ then add the ones/units together. Recombine to give the answer.'

Then ....

76
47
13 (7+6)
$110(70+40)$
123

Add the least significant digits (units) together first and then the tens in preparation for the written method.

If children are ready introduce the formal written method

23
15
$+\quad 38$
Then moving onto 'carry" across the columns and bridge 10 or 100.
$76+47=$
76
77
$+\quad 47$
123
11

Use the language of place value to ensure understanding:
"Seven add six equals 13". Write three in the units column and 'carry' one (10) across into the tens column. 'Add 40 and 70 and then 10 that we 'carried" equals 120'. Write 20 in the tens column and 'carry' one (100) across into the hundred column (100).

The digits that have been 'carried' should be recorded under the line in the correct column.

Further develop with addition of a three- digit number and a two-digit number and three-digit number and a three-digit number.

| $178+43=$ | $686+549$ |
| :---: | :---: |
| 178 | 686 |
| $+\frac{43}{221}$ | +549 |
| 11 | 1235 |
| 11 |  |

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Addition - Year 4

- Add numbers with up to 4 digits using formal written method of columnar addition where appropriate.

NB Ensure that children are confident with the methods outlined in the previous years' guidance before moving on.

Continue to teach the use of empty number lines with three and four digit numbers, ass appropriate.

Further develop the formal written method of addition, with three-digit numbers. Revise the expanded method first, if necessary:

$$
176+147=
$$

$+147$
$13(7+6)$
$110(70+40)$
$200(100+100)$ 323

This will lead into the formal written method....
$176+147=$
Use the language of place value to ensure understanding:
'Seven add six equals 13 . Write three in the units column and 'carry'
$+176$
323
11 one across into the tens column (10). '40 add 70 and the ten we carried equals 120.' Write 2 in the tens column (20) and carry 1 across into the hundreds column (100). '100 and 100 and 100 that has been carried equals 300.' Write 3 in the hundreds column (300).

The digits that have been 'carried' should be recorded under the line in the correct place value column.

If children are confident, introduce the addition of a four-digit number and a 3-digit number:
$1845+526=$

1845
+526
2371
11

Continue to develop with addition of two four-digit numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year 5

- Add whole numbers with more than 4 digit, including formal written method (columnar addition)

NB Ensure that children are confident with the methods outlines in the previous years guidance before moving on.

Continue to teach the use of empty number lines with larger numbers (and decimals), as appropriate.
Continue to develop the formal written methods for addition with larger numbers (add decimals numbers) and with the addition of three or more numbers

$$
21848+1523=
$$

21848
$+1523$
23371
11
Continue to use the language of place value to ensure understanding. Ensure that the digits have been 'carried" are recorded under the line in the correct place value column.

By the end of year 5 and into year 6 children should be using the formal written method for the addition of decimal numbers:

$$
£ 154.75+£ 233.82=
$$

154.75

Continue to use the language of place value to ensure understanding.
$+\quad 233.82$
388.57

1

Ensure that the decimal points line up.

Continue to practice and apply the formal written method throughout Year 5.

NB.. If, at any time, children are making significant errors, return to the previous stage in calculation.

## Addition - Year 6

No objectives have been included in the National Curriculum program of study explicitly related to the written methods for addition in Y 6 . However, there is an expectation that children will continue to practice and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance for methods).

During the first term Big maths continue to develop their methods of using formal written methods of addition to add a variety of decimals, including numbers with mixed amounts of decimal places.

Our aim is that by the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Stages in Subtraction

Teaching for Mastery FS/KS1 Subtraction example CPA

| Strategies | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Taking away ones <br> Use physical objects to demonstrate how something can be taken away. Move on to crossing out drawn representations. This can be developed by representing a group of ten with a line and ones with dots. | $5-1=$ |  | $\begin{aligned} & 18-3=15 \\ & 8-2=6 \end{aligned}$ <br> There are 15 cakes in the shop. One cake is eaten, how many are left. |

## Subtraction - Foundation Stage

- Say a number which is one less than a given number
- Use quantities and objects to subtract 2 single digit numbers by counting back

Children need to have a secure knowledge of number in order to begin subtraction and are then introduced to the concept of subtraction through practical games and activities.
Children act out subtractions to physically subtract a number of objects from a group. They use arm gestures to represent the signs - and =. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge by learning that subtracting means taking away a certain number of objects from a group (leaving them with fewer objects).
Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "subtraction means we take away objects from a group / we have 11 fewer objects now. Equals means we find out how many we have got left. We have only got 3 left!" Adults support children in recording their subtractions in the written form on whiteboards and in their maths books.

They will begin to relate subtraction to 'taking away' using objects to count 'how many are left' after some have been taken away.

$$
6-2=4
$$

'Take two apples away. How many left?'
Children will begin to count back from a given number.


## Subtraction - Year One

- Given a number, identify one less
- Read, write and interpret mathematical statements involving subtraction(-) and the equal (=) sign
- Subtract one-digit and twoOdigit numbers within 20, including zero
- Solve missing number problems e.g. $20-\square=15$

NB Ensure that children are confident with the methods outlined in the previous years' guidance before moving on.

Children will continue to practice counting back from a given number using number tracks and number lines

## 1)2345678910

$9-5=4$
'Put your finger on number nine. Count back five.'
Continue progression to a marked number line:
$12-6=6$

'Put your finger on number twelve and count back 6 .

$$
14-5=9
$$


'Put your finger on fourteen and count back five."

NB Ensure children are confident using a marked number line before moving to an empty number line (see year 2 guidance).
Continue practise counting back for subtraction with numbers within 20.

Counting on to find a difference:
Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of 'difference'.


Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

$$
11-9=2
$$



The difference between nine and eleven is two.
NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year Two

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
- A two digit number and ones
- A two digit number and tens
- Two two-digit numbers

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Counting back using an empty number line within 100, in one ...

$$
34-6=28
$$


and in tens:

$$
58-30=28
$$



Use in conjunction with a 100 square to show jumps of tens.


Subtraction, using partitioning, on an empty number line:


Use in conjunction with a 100 square to show jumps of tens and ones.

If children are confident, use more efficient jumps:

$$
76-45=31
$$



$$
76-40-5=31
$$

Use in conjunction with a 100 square to show jumps of tens and ones.

Counting on to find a small difference:

Introduce complementary addition to find differences (only use for small differences). The use of model is extremely important here to understand the idea of 'difference' (see Y 1 guidance).

$$
12-8=4
$$


'The difference between 8 and 12 is 4.'
$32-28=4$

' The difference between 28 and 32 is 4.'

If children are confident, further develop this method:

$$
76-58=18
$$


'The difference between 58 and 76 is 18.'

Further develop subtraction with numbers that bridge 100, using a 200 grid to support.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year Three

- Subtract numbers with up to three digits, using formal written methods of columnar subtraction

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the use of the empty number line with calculations that bridge 100:
$126-45=81$


Then use more efficient jumps:


Extend with larger numbers by counting back ...

$$
216-27=189
$$


... and by counting on to find the difference (small difference):

$$
231-198=33
$$


'The difference between 198 and 231 is 22.'

Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns). Use two-digit numbers when introducing this method, initially:

| 78-23 $=55$ | 'Partition numbers into tens and ones/units. Subtract the ones, and <br> then subtract the tens. Recombine to give the answer.' |
| ---: | :--- |
| $70+8$  <br> $-20+3$ NB In this example decomposition (exchange) is not required. |  |

You might replace the + sign with the word 'and' to avoid confusion, or omit completely.
This will lead to the formal written method:
$\begin{array}{ll}78 & \text { Use the language of place value to ensure understanding: } \\ -\quad 23 & \text { 'Eight subtract three, seventy subtract twenty.' }\end{array}$

NB A number line would be an appropriate method for this calculation but use two-digit numbers to illustrate the formal written method initially.

Introduce the expanded written method where exchange/decomposition is required:

| $73-27=$ |  |  |
| :---: | :---: | :---: |
|  | Becomes |  |
| $70+3$ |  |  |
| $-\quad$ |  | $60+13$ |

$$
73 \text { is partitioned into } 60+13 \text { in }
$$

order to calculate 72-27

NB children will need to practise partitioning numbers in this way. Base - ten materials could be used to support this.

When children are confident with the expanded method introduce the formal written method, involving decomposition/exchange:

$$
73-27=
$$

613
Use the language of place value to ensure understanding.
73 'We can't subtract seven from three, so we need to exchange a ten

- 27 for ten ones to give us $60+13$.' Use base ten material to support understanding.

Then they use the same methods as above extending to 3-digit number subtract a 2-digit number and $a$ three-digit number subtract $a$ three-digit number. NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year Four

- Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate.

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teacher the use of empty number lines with three and four digit numbers, as appropriate.

Continue to develop the formal written method of subtraction by revisiting the expanded method first, if necessary. Continue to use base ten materials to support understanding.

$$
\begin{array}{rc}
200+50+8 \\
70+3
\end{array} \quad \text { becomes } \begin{gathered}
100+150+8 \\
\\
\end{gathered} \begin{aligned}
& -70+3 \\
& \hline 100+80+5=185
\end{aligned}
$$

You might replace the + sign with the word 'and' or omit it altogether to avoid confusion. Children will need to practise partition in a variety of ways.

This leads to the formal written method, involving decomposition...

115
258

| 73 |
| :--- |
| 175 |\(\quad \begin{aligned} \& In this example it has been necessary to exchange from the <br>

\& hundred column.\end{aligned}\)
Use the language of place value to ensure understanding.

The children's understanding is developed further using these methods to subtract a 3-digit number from a three-digit number. When children are confident, develop with four-digit numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year 5

- Subtract whole numbers with more than 4 digits, including formal written method (columnar subtraction)
- Subtract numbers mentally with increasingly large numbers
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

NB Ensure that children are confident with the methods outline in the previous years' guidance before moving on.

Continue to teach the use of empty number lines with larger numbers and decimals, as appropriate.

Continue to develop the formal written method for subtraction with three and four digit numbers (see Y4 guidance), returning to an expanded method and using base ten materials, is necessary. When children are confident with previous stages extend with larger numbers (and decimal numbers). Return to an expanded method, if necessary.

```
            612 11
12731
\begin{array} { l } { 1 2 7 3 6 1 } \\ { - \quad 1 ~ 3 ~ 6 ~ 7 } \\ { \hline 1 4 ~ 3 ~ 6 4 } \end{array}
                                    In this example it has been necessary to exchange from the tens and the hundreds columns.
```

NB.. If children are making significant errors; provide calculations where only one exchange is required.

Continue to practice and apply the formal written methods with large numbers throughout year five.

NB.. If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year 6

- Perform mental calculations, including mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations and determine in the context of a problem, an appropriate degree of accuracy
- Subtract fractions with different denominations and mixed numbers, using the concept of equivalent fractions

Non-statutory guidance for the Program of Study related to written methods for subtraction in Year 6, suggest that pupils practice and use the formal written methods of columnar subtraction for larger numbers and decimals and use these methods when solving problems, as appropriate (see previous years' guidance for methods).

Introduce subtraction of decimals, initially in the context of money and measures.
£166.25-£ $83.72=$

|  | 16 | 5 | 12 |  |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 6 | 6 | 2 | 5 |
| - | 8 | 3. | 7 | 2 |
|  | 8 | 2. | 5 | 3 |

Continue to practice and apply the formal written methods with large numbers and decimals throughout year five.

Our aim is that by then end of Y 6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Stages in Multiplication

## Multiplication - Foundation Stage

- Solve problems, including doubling

By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through discussion, practical games and activities, including the use of songs, rhymes and activities indoors and in the outdoor area.
Children act out 'doubling' by physically adding two equal groups together to find out the 'doubles' answer.

'Three apples for you and three apples for me. How many apples altogether?'

## Multiplication - Year One

- Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Count in multiples of twos, fives and tens (to the $10^{\text {th }}$ multiple)

Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts. They will solve practical problems that involve combining groups of 2,5 or 10 e.g. socks, fingers and cubes.

'Six pairs of socks. How many socks altogether?'
' $2,4,6,8,10,12$

'Three packs of 10 crayons. How many crayons altogether?'

$$
'
$$

Use arrays to support early multiplication..

'Five groups of two faces. How many faces altogether?'
' $2,4,6,8,10$ '
'Two groups of five faces. How many altogether?'
'5, 10'

'2 groups of 5'
'How many altogether?'
$' 5+5=10 '$
'Double 5 is $10 '$

Continue to solve problems in practical contexts and develop the language of early multiplication, with appropriate resources throughout Y 1 .

| Arrays showing commutative multiplication <br> Pupils should understand that an can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. | $\begin{aligned} & 3 \times 5=15 \\ & 5 \times 3=15 \\ & 15 \div 3=5 \\ & 15 \div 5=3 \end{aligned}$ | Draw arrays in different rotations to find commutative multiplication sentences. | $\begin{aligned} & 0000^{4 \times 2-8} \\ & 0000^{2 \times 4-8} \\ & 00 \quad 2 \times 4=8 \\ & 00 \\ & 00 \\ & 4 \times 2=8 \\ & 08 \\ & 00 \\ & 00 \\ & 4 \times 2=8 \end{aligned}$ | 3 children go to the park to hunt for plne cones. They find 5 each, how many do they find altogether? <br> 5 children eat the same number of cakes at a party. 15 cakes are eaten in total, how many did they each eat? $\begin{aligned} & 5+5+5=15 \\ & 3 \times 5=15 \\ & \\ & 3+3+3+3+3=15 \\ & 5 \times 3=15 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |

## Multiplication - Year 2

- Recall and use multiplication facts for the 2,5 and 10 multiplication tables
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication ( $x$ ) and equal (=) signs
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts
- Show that multiplication of two numbers can be done in any order (commutative)

NB.. Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the $x$ sign to record.
Combining Groups (repeated addition):

'3 groups of 10 crayons'
'How many crayons altogether?'
$" 10+10+10=30$ '
'3 groups of 10'
'3 times 10'
$' 3 \times 10=30$ '

'5 groups of 3 '
'3+3+3+3+3=15'
'3 multiplied by 5'
$' 3 \times 5=15$ '

$$
6 \times 5=30
$$


' $5+5+5+5+5+5=30$ '
' 6 rows of 5'
'6 groups of 5'
'5 groups of 6'
' $5 \times 6=30$ '
$' 6 \times 5=30$ '

Use an empty number line:
$6 \times 5=30$


NB.. If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Year 3

- Recall and use multiplication facts for the 3,4,5 and 8 multiplication tables (continue to practices the 2,5 and 10 multiplication tables)
- Write and calculate mathematical statements for multiplication using the multiplication table that they know, including for 2-digit number lines one-digit numbers, using mental and progressing to a formal written method

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.
Continue to use number lines and arrays to support multiplication, as appropriate (see Y2 guidance).

$$
4 \times 3=12
$$



Partitioning method for multiplication of a teens number by a one-digit number.
$13 \times 5=65$ (partition 13 into 10 and 3)
$10 \times 5=50$
$3 \times 5=15$
$50+15=65$

Demonstrate the partitioning method using a number line:
$13 \times 5=65$
$10 \times 5=50$

$$
3 \times 5=15
$$



Grid Method (teen number multiplied by a one-digit number):
$13 \times 8=104$

| $x$ | 10 | 3 |
| :---: | :---: | :---: |
| 8 | 80 | 24 |

$80+24=104$
'Partition 13 into $10+3$ then multiply each number by 8. Add the partition products ( 80 and 24 ) together.'

This will lead into expanded short multiplication:
$13 \times 8=104$
$10+3$
Include an addition symbol when adding partial products.
$\times 8$
$24(3 \times 8)$
$+80(10 \times 8)$
10

Refine the recording in preparation for formal short multiplication:
$13 \times 8=104$
Use the language of place value to ensure understanding.
13
$\times 8$ Include an addition symbol when adding partial products.
$24(3 \times 8)$
$+80(10 \times 8)$
104

Model the same calculation using a number line, if necessary, to ensure understanding.

Formal short multiplication:

13
$\times 8$
104
2

Ensure that the digit 'carried over' is written under the line in the correct column.

Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout year 3 using teen numbers by a one-digit number.

If children are confident progress to multiplying other two - digit numbers by a one-digit number (see year 4 guidance)

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Multiplication - Year 4

- Recall multiplication facts for multiplication tables up to $12 \times 12$
- Multiplication two-digit and three-digit numbers by a one-digit number using formal written layout
- 

NB Ensure that the children are confident with the methods outline in the previous year's guidance before moving on.

Continue to use empty number lines, as appropriate (see year 3 guidance).

Further develop the grid method for two-digit numbers multiplied by a one- digit number.
$36 \times 4=144$

| $X$ | 30 | 6 |
| :---: | :---: | :---: |
| 4 | 120 | 24 |

$120+24=144$ (add the partial products)

Expanded short multiplication (†wo - digit number by a one-digit number):
$36 x=144$
$30+6$
$\times 4$ Include an addition symbol when adding partial products.
$24(4 \times 6=24)$
$+120(4 \times 30=120)$
144

Refine recording in preparation for formal short multiplication:
$36 \times 4=144$
36
$\times 4$
$+24(4 \times 6)$
$120(4 \times 30)$
144
28

This leads to short multiplication (formal method) of a two-digit number multiplied by a one - digit number:

$$
\begin{array}{cl}
36 \\
\times 4 \\
\hline 144
\end{array} \quad \begin{aligned}
& \text { Use the language of place value to ensure understanding. } \\
& \frac{\text { Ensure that the digit 'carried over' is written under the line in the }}{2}
\end{aligned}
$$

Continue to practise the formal method of short multiplication of a two-digit number by a onedigit number throughout $Y 4$.

If children are confidence, continue to develop short multiplication with three-digit numbers multiplied by a one-digit number.

If necessary, return to the grid method and/or expanded method first:

$$
127 \times 6=762
$$

| $X$ | 100 | 20 | 7 |
| :---: | :---: | :---: | :---: |
| 6 | 600 | 120 | 42 |

$600+120+42=762$ (add the partial products)

This leads to expanded short multiplication:

$$
\begin{aligned}
& 127 \times 6=762 \\
& 127 \\
& \times 6 \\
& \hline 42(6 \times 7) \\
& 120(6 \times 20) \\
& +600(6 \times 100) \\
& \hline 762
\end{aligned}
$$

This will lead into short multiplication (formal method):

## 127

$\times 6$ Use the language of place value to ensure understanding.
762
Ensure that the digits 'carried over' are written under the in the correct column.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Multiplication - Year 5

- Multiply numbers up to a 4 digits by one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

NB Ensure that children are confident with the method outline in the previous year's guidance before moving on.

Build on the work covered in Y4 with the formal method of short multiplication (two-digit number multiplied by a one-digit number).

When children are confident introduce multiplication by a two-digit number. If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a teen number):
$23 \times 13=(20+3) \times(10+3)=299$

| $X$ | 20 | 3 |
| :---: | :---: | :---: |
| 10 | 200 | 30 |
| 3 | 60 | 9 |

$$
\begin{array}{r}
230 \\
+69 \\
\hline 299
\end{array}
$$

Add the partial products $(200+30)+(60+9)=299$

Expanded long multiplication (two-digit numbers multiplied by a teen number):

$$
\begin{gathered}
23 \\
\times 13 \\
\hline 9(3 \times 3) \\
60(3 \times 20) \\
30(10 \times 3) \\
200(10 \times 20) \\
\hline 299
\end{gathered}
$$

This leads into ....

Compact long multiplication (formal method):
$23 \times 13=299$

23
$\times 13$
$69(3 \times 23)$
$230(10 \times 23)$

Use the language of place value to ensure understanding.

Add the partial products.

Extend to larger numbers.

When the children are confident with long multiplication extend with three-digit number by a twodigit number, returning to the grid method first if necessary. (Refer to Y6)

Multiplication - Year 6

- Multiply multi-digit numbers (including decimals) up to 4 digits by two-digit whole numbers

NB Ensure that children are confident with the methods outline in the previous year's guidance before moving on.

Continue to practise and develop the formal short multiplication method and formal long multiplication method with larger numbers and decimals throughout Y 6 . Return to an expanded form of calculation initially if necessary (see Y 5 guidance)

Compact long multiplication (formal method)


The grid method (decimal number multiplied by a two- digit number):

$$
53.2 \times 24=1276.8
$$

| $X$ | 50 | 3 | 0.2 |  |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 1000 | 60 | 4 | 1064.0 |
| 4 | 200 | 12 | 0.8 | 212.8 |
|  |  |  |  | 1276.8 |

The formal written method of long multiplication:

| 53.2 |
| ---: |
| $\times 24.0$ |
| $21^{1} 2.8$ |
| 1064.0 |
| 1276.8 |

It is an option to include .0 in this example, but not essential.
(53.2 $\times 4$ )
(53.2 x 20)

The prompt (in brackets) can be omitted if the children no longer need them.

NB.. If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y 6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

## Stages in Division

| Strategies | Concrete $\square$ | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
|  |  |  | Share 9 buns between three people. $9 \div 3=3$ <br> Can you make up your own 'sharing' story and record a matching equation? |

## Division - Foundation Stage

- Solve problems, including halving and sharing
- 

By the end of Reception, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through discussion, songs and rhymes, practical games and activities. They act out 'halving and sharing' through activities such as sharing food within the context of role play, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects.

'Share the apples between two people.'
'Half of the apples for you and half of the apples for me.'

## Division - Year 1

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Count in multiples of twos, fives and tens (to the $10^{\text {th }}$ multiple)

Children will start with practical sharing using a variety of resources. They will share objects into equal groups in a variety of situations. They will begin to use the vocabulary associated with division in practical contexts.
'Share these eight apples equally between two children. How many apples will each child have?'

'Share 20 crayons between 2 pots.'

How many crayons are in each pot?'

Children will move from sharing to grouping in a practical way

| Division as |  | Show jumps in groups. The number of jumps equals the number of groups. | $28 \div 7=4$ |
| :---: | :---: | :---: | :---: |
| grouping <br> Here, division is shown as grouping. If we have ten cubes and put them into groups of two, there are 5 groups. This is a good opportunity to demonstrate and reinforce the inverse relationship with multiplication. | Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. | Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. | Divide 28 into 7 groups. How many are in each group? <br> Max is filling party bags with sweets. He has 20 sweets altogether and decides to put 5 in every bag. How many bags can he fill? |


'Put 20 crayons into groups of 10. How many pots do we need?'

Use arrays to support early division .....


'How many faces altogether?' 'How many groups of 2?'

"How many groups of 5?'
'10 shared equally between 2 people.'
'Half of ten is five"

Continue to solve problems in practical contexts throughout y 1 , and develop the language of early division, with appropriate resources.

## Division - Year 2

- Recall and use multiplication and division facts for the 2,5 and 10 multiplication tables
- Calculate mathematical statements for division within the multiplication tables they know and write them use the division $(\div)$ equals $(=)$ signs
- Solve problems involving division, and multiplication and division facts, including problems in contexts

NB... Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.
Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the $\div$ sign to record, using multiples that they know.

Sharing and grouping:

' 30 crayons shared equally between three
pots' (Sharing)
'We have 30 crayons and put ten crayons in each pot. How many pots do we need?' (Grouping)
' 30 divided by $10=3$ '
$30 \div 10=3$
' 30 divided by $3=10$ '

$15 \div 5=3$
$15 \div 3=5$
'How many groups of 5?'
'15 shared equally between 3 people is ....?'
' 15 divided by 3 equals 5 '
'15 divided by 5 equals 3'

Using arrays to support division

$$
15 \div 5=3
$$

$$
15 \div 3=5
$$



How many groups of 3?
How many groups of 5 ?
15 shared between 3 people is ...?
15 shared between 5 people is....?

15 divided by $5=3$
15 divided by $3=5$

When children are ready, use an empty number line to count forwards:

$$
30 \div 5=6
$$

'How many jumps of five make thirty?'


Also jump back to make the link with repeated subtraction:

$$
30 \div 5=6
$$

'How many groups of five?'


NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Division - Year 3

- Recall and use multiplication and division facts for the 3, 4, and 8 multiplication tables (continue to practice the 2, 5, and 10 multiplication tables)
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method.

NB Ensure that children are confident with the methods outline in the previous year's guidance before moving on.
Continue to use practical resources, pictures, diagrams, number lines, arrays and the $\div \operatorname{sign}$ to record, using multiples that they know, as appropriate (year 2 guidance).

Use empty number lines to count forwards...
$24 \div 3=8$
'How many threes in 24?'

$\begin{array}{lllllllll}0 & 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24\end{array}$
... also jump back from 24 to make the link with repeated subtraction.

'How many groups of three in 24?'

Introduce the formal layout using multiplication/division facts that the children know: $24 \div 3=8$

This can also be recorded as ...

'Twenty four divided by three equals eight.'
'How many threes are there in twenty four?'

NB..If, at any time, children are making significant errors, return to the previous stage in calculation.

## Division - Year 4

- Recall multiplication and division facts for multiplication tables up to $12 \times 12$
- Use place value, know and derived facts to divide mentally
- Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stage in the programs of study but implied in the non-statutory guidance)

NB Ensure that children are confident with the methods outline in the previous year's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

$$
32 \div 8=4
$$

Continue using the formal written layout for division using multiplication tables that they know:

'How many eights are there in thirty two?'

NB Remainders are not specifically referred to until Y 5 in the national Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts...

Continue using the formal written layout, introducing remainders:

$$
25 \div 3=8 r 1
$$

8 r 1
3
25 This could be modelled using an empty number line, if necessary. Alternatively you could jump forwards in multiples of 3 from 0 to 24 ('and one more makes 25')


Division using partitioning (two digits divided by one digit):
$65 \div 5=13$
$65=50+15$ (Partition 65 into 50 and 15)
$50 \div 5=10$
$15 \div 5=3$
$10+3=13$

NB Children will need to practice partition in a variety of ways.

Written method leading to formal method:
$98 \div 7=14$
'We have partitioned 98 into 70 and $28(90=70$
 + 28)'.
Seven 'goes into' 70 ten times and seven 'goes into' 28 four times. Ten add four equals 14

This will lead to the formal written method of short division:
$98 \div 7=14 \quad$ Use the vocabulary of place value to ensure understanding and make the link to partitioning.

$$
\begin{gathered}
14 \\
7 \longdiv { 9 ^ { 2 } 8 }
\end{gathered}
$$

Continue to practise the formal method of short division throughout $Y 4$.

When children are confident develop further to 3 and 4 digit numbers divided by a one-digit number using the formal method of short division with whole number answers (no remainders)

NB.... If, at any time, children are making significant errors, return to the previous stage in calculation.

## Division - Year 5

- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
NB Ensure that the children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise the formal written method of short division with whole number answers....
$184 \div 8=23$
23
$8 \longdiv { 1 8 ^ { 2 } 4 }$

Use the language of place value to ensure understanding.

Make links to the partitioning method (see Y4 guidance).
and with remainders...
$432 \div 5=86 r 2$

86 r 2
5
$43^{3} 2$

The remainder can also be expressed as a fraction $2 / 5$ (the remainder divided by the divisor):
$432 \div 5=862 / 5$

Continue to practice, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the contexts.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

## Division - Year 6

- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context

NB...Ensure that children are confident with the methods outlined in previous year's guidance before moving on.

Continue to practice the formal method of short division, with and without remainders, using the language of place value to ensure understanding (see $У 5$ guidance).

Short division:
$496 \div 11=45 \mathrm{r} 1$

45 r 1
11
$49^{5} 6$

The remainder can also be expressed as a fraction $1 / 11$ (the remainders divided by the divisor)
Dividing a two-digit number using formal methods of long division:

| 45 r 1 | Multiples of the divisor (11) have been subtracted |
| :---: | :---: |
| $1 1 \longdiv { 4 9 6 }$ | from the dividend (496) |
| $-\frac{440}{56}(40 \times 11)$ | '40 (lots of 11) $+5($ lots of 11$)=45^{\prime}$ |
| - 5 5 $(5 \times 11)$ | '1 is the remainder' |
| 1 (remainder) | Answer 45 1/11 |

Standard short division does not help with the following calculation. However, it can be solved using long division (by repeated subtraction using multiples of the divisor):
$144 \div 16=9$

| 9 | Multiples of the divisor (16) have been subtracted from the dividend (144) |
| :---: | :---: |
| $\begin{aligned} & 1 6 \longdiv { 1 4 4 } \\ & -64(4 \times 16) \end{aligned}$ |  |
| $\begin{array}{r} 80 \\ -64(4 \times 16) \end{array}$ | '4 (lots of 16$)+4($ lots of 16$)+1($ lot of 16$)=9$ (lots of 16) |
| $\begin{array}{r} 16 \\ -16(1 \times 16) \\ \hline \end{array}$ |  |
| 0 | There is no remainder |

Children will need to select the most effective method fro each calculation/problem they meet, including whether to use the standard, formal written method of long division:

```
432\div15=28 r12
15 
    132
    120(8\times15)
    12 (remainder)
Multiples of the division (15) have been subtracted from the dividend (432)
\(' 20(\) lots of 15\()+8(\) lots of 15\()=28\)
12 is the remainder
```

The remainder can also be expressed as a fraction, (the remainder divided by the divisor) or as a decimal, 0.8 (see next example)

The answer is: $2812 / 15$ or 28.

This is an alternative way of recording formal long division:

```
432 * 15=28.8
            28.8
    15\longdiv{432.0}
        30
        132
        120
            120
            12 0
```

                                NB Only teach this method when children are
                                completely secure with the pervious method.
    The remainder is expressed as a decimal

NB If, at any time, children are making significant errors, return to the previous stage in calculation.
Our aim is that by the end of Y 6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

